Action canceled

Internet Explorer was unable to link to the Web page you requested.
The page might be temporarily unavailable.

Complex number raised to a complex power

Download Zeno 1.2

by Stephen R. Schmitt

complex base$^{\text{complex exponent}}$

x =		+		j
y =		+		j
x^y =		+		j

Compute

Contents

1. About

This Java Script calculator raises a complex number to a complex power. To operate the calculator, enter the real and imaginary parts of the base number and the exponent. Press the **Compute** button to obtain the solution. On invalid entries, a popup window will display an error message.

Return to Contents

2. The source code

The Java Script source code for this program can be viewed by using the **View|Source** command of your web browser.

You may use or modify this source code in any way you find useful, provided that you agree that the author has no warranty, obligations or liability. You must determine the suitability of this source code for your use.

Return to Contents

3. Discussion

A complex number is defined as:

$$z = a + j \cdot b$$

where a and b are real numbers and $j^2 = -1$. For the imaginary number j, we can verify that j^j is a real number. To show this, recall Euler's formula:

$$e^{j \cdot \theta} = \cos(\theta) + j \cdot \sin(\theta)$$

where e is the base of natural logarithms. Now let $\theta = \pi/2$. Since

$\cos(\pi/2) = 0$ and
$\sin(\pi/2) = 1$

we get

$$j = e^{j\pi/2}$$

Then, using some algebra

$$j^j = (e^{j \cdot \pi/2})^j = e^{j \cdot j \cdot \pi/2} = e^{-\pi/2},$$

which is equal to 0.20787957635 and is a real number.

A derivation

We can use Euler's formula to raise a real number to a complex power. Any real number r can be written as $e^{\ln r}$ - so the complex power of a real number is:

$$\begin{aligned} r^{j \cdot z} &= (e^{\ln r})^{j \cdot z} \\ &= e^{(j \cdot z \cdot \ln r)} \\ &= \cos(z \cdot \ln r) + j \cdot \sin(z \cdot \ln r) \end{aligned}$$

To raise a complex number $x = a + j \cdot b$ to the complex power $y = c + j \cdot d$ we can now do the following:

$$x^y = (a + j \cdot b)^{c + j \cdot d}$$

Using Euler's formula, let

$$a + j \cdot b = \rho \cdot e^{j \cdot \theta}$$

where,

$$\rho = \sqrt{(a^2 + b^2)}$$
$$\theta = \arctan(b/a)$$

By substitution, we get

$$x^y = (\rho\cdot e^{j\cdot\theta})^{c}\cdot(\rho\cdot e^{j\cdot\theta})^{j\cdot d}$$

Expanding, we get

$$\begin{aligned}
&= \rho^{c}\cdot e^{j\cdot c\cdot\theta}\cdot\rho^{j\cdot d}\cdot e^{j\cdot j\cdot d\cdot\theta} \\
&= \rho^{c}\cdot e^{-d\cdot\theta}\cdot e^{j\cdot c\cdot\theta}\cdot\rho^{j\cdot d} \\
&= \rho^{c}\cdot e^{-d\cdot\theta}\cdot[\cos(c\cdot\theta) + j\cdot\sin(c\cdot\theta)]\cdot[\cos(d\cdot\ln\rho) + j\cdot\sin(d\cdot\ln\rho)] \\
&= \rho^{c}\cdot e^{-d\cdot\theta}\cdot\{[\cos(c\cdot\theta)\cdot\cos(d\cdot\ln\rho) - \sin(c\cdot\theta)\cdot\sin(d\cdot\ln\rho)] + \\
&\qquad j\cdot[\cos(c\cdot\theta)\cdot\sin(d\cdot\ln\rho) + \sin(c\cdot\theta)\cdot\cos(d\cdot\ln\rho)]\} \\
&= \rho^{c}\cdot e^{-d\cdot\theta}\cdot[\cos(c\cdot\theta + d\cdot\ln\rho) + j\cdot\sin(c\cdot\theta + d\cdot\ln\rho)]
\end{aligned}$$

To summarize, given

$$x = a + j\cdot b$$
$$y = c + j\cdot d$$

Then

$$x^y = \rho^{c}\cdot e^{-d\cdot\theta}\cdot[\cos(c\cdot\theta + d\cdot\ln\rho) + j\cdot\sin(c\cdot\theta + d\cdot\ln\rho)]$$

where,

$$\rho = \sqrt{(a^2 + b^2)}$$
$$\theta = \arctan(b/a)$$

Return to Contents

Free-CDSoftware | Academic Superstore | 256MB PC133 | Easy. Fast. Cheap. | Get a FREE 2-week trial from Netflix!

Copyright © 2004, Stephen R. Schmitt